MATH 11th grade

The Normal Distribution

1. 300 contestants enter a swimming competition. The 60 contestants with the fastest times go through to the semi-finals.

(a) Calculate the percentage who go through to the semi-finals. *[1 mark]*

The times are approximately normally distributed with a mean of 204 seconds and a standard deviation of 6 seconds.

(b) Represent this information on a normal distribution graph, indicating clearly the mean and percentage who reach the semi-finals. *[3 marks]*

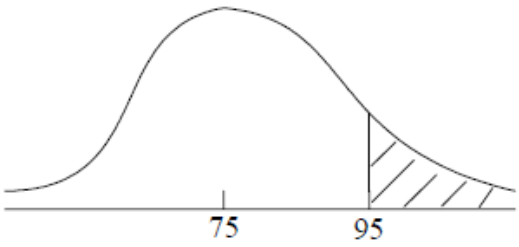
(c) Calculate the time of the slowest contestant to reach the semi-finals. *[4 marks]*

(Total 8 marks)

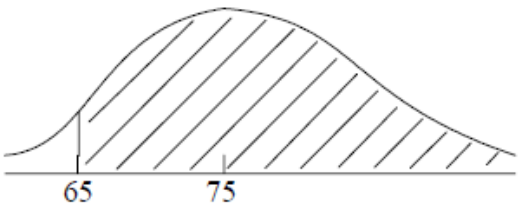
2. A set of 1000 test scores is normally distributed, with a mean of 75 and a standard deviation of 10.

(a) Calculate the probability which is represented by each of the following diagrams, giving your answers to 3 decimal places.

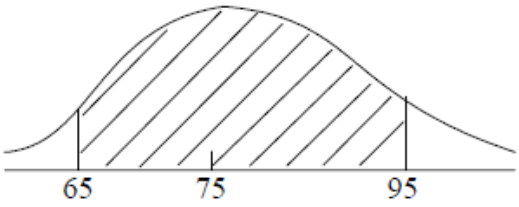
(i)



(ii)



(iii)



[6 marks]

(b) Out of the thousand students, how many received test results higher than 87? *[3 marks]*

(Total 9 marks)

3. Speed checks on a very large number of cars at a certain point on a motorway show that the speeds are normally distributed.

10% of cars have speeds more than 132.8 km h^{-1} and only 7% of cars have speeds less than 103.2 km h^{-1}.

(a) Draw a normal distribution diagram to illustrate this information, indicating clearly both the percentages and the speeds. *[2 marks]*

It is established that the mean speed of the cars is 119 km h^{-1}, while the standard deviation of the speeds is 10.7 km h^{-1}.

(b) Calculate the percentage of cars travelling at more than 130 km h^{-1}. *[2 marks]*

(c) The slowest 40% of the cars travel at a speed less than s km h^{-1}. Find s. *[2 marks]*

(Total 6 marks)

4. (i) A variable X follows a normal distribution with a mean of -5 and standard deviation of 9.

Find the probability that a randomly chosen item from this population has a negative value. *[2 marks]*

(ii) A firm delivers milk to a number of homes in a street. The time taken to complete these deliveries follows a normal distribution with mean of 10 minutes and standard deviation 1.5 minutes. Milk is delivered every day of the week.

(a) What percentage of the days of the year would you expect the milk deliveries to take 10 minutes or less? *[1 mark]*

(b) (i) On a suitable diagram, show the position of the mean time taken, the position of one positive standard deviation from the mean, and the 12 minute mark.

(ii) On your diagram, shade the area under the curve that represents the number of days that the milk deliveries take more than 12 minutes.

(iii) In a period of 200 days, calculate the number of days when the milk deliveries take between 11.5 and 12 minutes. *[6 marks]*

(c) Find how many minutes are needed for the fastest 5 % of deliveries. *[2 marks]*

(Total 11 marks)